DOES $\sum_{n=1}^{\infty} \frac{n}{2n-1}$ CONV / DIV ?

$f(x) = \frac{x}{2x-1} > 0$, CONT, DECR ON $[1, \infty)$

$f'(x) = \frac{1(2x-1) - x(2)}{(2x-1)^2} = \frac{-1}{(2x-1)^2} < 0$

CAN USE $\int$ TEST

$\int_1^{\infty} \frac{x}{2x-1}\,dx = \ldots$

BUT

$\lim_{n\to\infty} \frac{n}{2n-1} = \lim_{n\to\infty} \frac{1}{2-\frac{1}{n}} = \frac{1}{2-0} = \frac{1}{2} \neq 0$

$\sum_{n=1}^{\infty} \frac{n}{2n-1}$ DIV BY DIV TEST

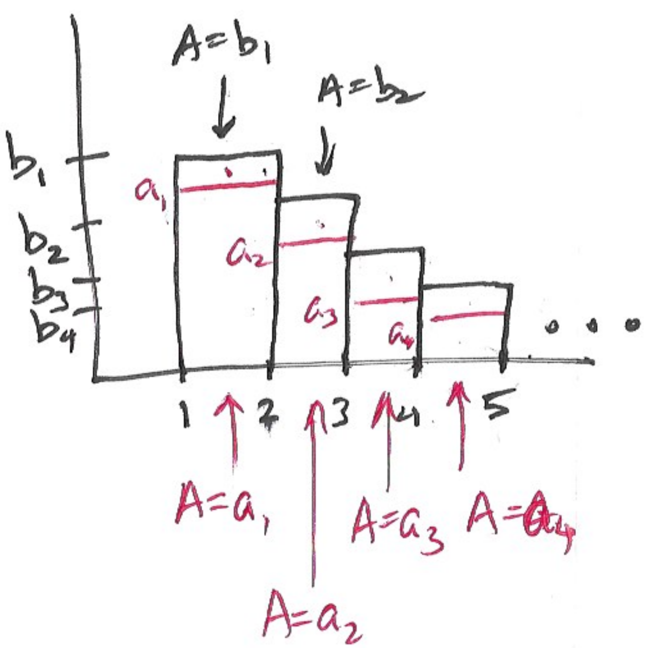
TESTS SO FAR

RUN TESTS WITH LEAST WORK FIRST

2. GEOMETRIC SERIES → ALWAYS (IF TEST APPLIES)
3. DIVERGENCE → ONLY TELLS IF SERIES DIV
4. INTEGRAL → ALWAYS (IF TEST APPLIES)
1. P-SERIES → ALWAYS A CONCLUSION (IF TEST APPLIES)

11.4 COMPARISON TEST

TH'M: IF $0 < a_n \le b_n$ FOR ALL $n \in \mathbb{Z}^+$ AND $\sum_{n=1}^{\infty} b_n$ CONV, THEN $\sum_{n=1}^{\infty} a_n$ CONV



SIMILAR TO COMPARISON TEST FOR IMPROPER INTEGRA[LS]

(7.8 ?)

TH'M: IF $0 < a_n \le b_n$ FOR ALL $n \in \mathbb{Z}^+$ AND $\sum_{n=1}^{\infty} a_n$ DIV, THEN $\sum_{n=1}^{\infty} b_n$ DIV

IF $0 < a_n \le b_n$ FOR ALL $n \in \mathbb{Z}^+$ AND $\sum_{n=1}^{\infty} a_n$ CONV OR $\sum_{n=1}^{\infty} b_n$ DIV THEN NO CONCLUSION IS POSSIBLE WITHOUT OTHER WORK.

DOES $\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$ CONV/DIV ?

P-SERIES
GEOMETRIC
DIVERGENCE
INTEGRAL

? COMPARISON

$$\underbrace{0 <}\ \overbrace{\frac{2}{n^2+2n} < \frac{2}{n^2}}$$

LARGEST TERM
IN DENOM
AS $n \to \infty$

LARGER DENOM, SAME NUMER
FRACTION IS SMALLER

$$\underbrace{\sum_{n=1}^{\infty} \frac{2}{n^2}} = \underbrace{2\sum_{n=1}^{\infty} \frac{1}{n^2}} \ \underbrace{\text{CONV}} \ (\underbrace{\text{p-SERIES TEST}})$$

$$\underbrace{p=2}\ \underbrace{>1}$$

SO, $\underbrace{\sum_{n=1}^{\infty} \frac{2}{n^2+2n} \text{ CONV}}$ $(\underbrace{\text{COMPARISON TEST}})$

DOES $\sum_{n=1}^{\infty} \frac{n+1}{3n^2-1}$ CONV/DIV ?

LARGER NUMER

$$\frac{n+1}{3n^2-1} > \frac{n}{3n^2} = \frac{1}{3n}$$

SMALLER DENOM

$$\cancel{\frac{n+1}{3n^2-1}} \qquad \cancel{\frac{1}{3n}}$$

LARGER NUMER

$$\frac{n+1}{3n^2-1} > \frac{n}{3n^2-1} > \frac{n}{3n^2}$$

SMALLER DENOM

$$0 < \frac{1}{3n} < \frac{n+1}{3n^2-1}$$

NOTE: $3n^2-1=0$ ONLY AT $n=\pm\frac{1}{\sqrt{3}}$

not $\geq n=1$

$\boxed{\text{TO BE CONTINUED}}$